

ΣΟΦΙΑ—SOPHIA

DOI: <http://dx.doi.org/10.18634/sophiaj.14v.2i.699>

Didactic engineering for learning the linear function through situations modeling*

Milton Cesar Campeón Becerra **

Eliecer Aldana Bermúdez***

Jhony Alexander Villa Ochoa ****

* Research carried out in order to obtain the title of Master's degree in mathematics teaching "Learning the function concept from the modeling of situations in context through didactic engineering". Group GEMAUQ (Mathematics Education Research Group of Universidad del Quindío). This project was financed with own resources of the authors.

** Master's degree in mathematics teaching, teacher at the Educational Institution Nuestra Señora de los Dolores (Quinchía Risaralda), GEMAUQ Group, 1000toncesar@gmail.com, Carrera 5 # 9 - 55 Riosucio Caldas.

*** PhD in mathematics education, full-time professor at Universidad del Quindío, Mathematics Education Research Group Coordinator (GEMAUQ), eliecerab@uniquindio.edu.co, Avenida Bolívar, Cra 15 Calle 12 Norte.

**** PhD in Education, Master's degree in Education: Teaching of Mathematics, Specialization in Education: Teaching of Mathematics, Bachelor of Education: Mathematics and Physics. Research professor at Universidad de Antioquia, jhonyvilla@gmail.com. Calle 67 # 53 - 108 Bl 9 of 415.

Información del artículo

Received: Marzo 2 de 2017

Revised: Agosto 21 de 2017

Accepted: Junio 30 de 2018

How cite:

Campeón, M.C., Aldana, E., Villa, J.A., (2018) Ingeniería didáctica para el aprendizaje de la función lineal mediante la modelación de situaciones. *Sophia*, 14 (2), 115-126.



ISSN (electrónico): 2346-0806 ISSN (impreso): 1794-8932



UNIVERSIDAD
La Gran Colombia

Fundada en 1951

Sophia-Educación, volumen 14 número 2. versión español

Resumen

In this article, we analyzed the way how students learn the concept of linear function from a didactic engineering, in which they have to develop modeling situations tasks in context. The main purpose is to promote the learning of the concept of function through modeling of situations, which is achieved through the confrontation between the a priori and a posteriori conceptions of the students in phase 4 of engineering. The methodology (that was) used, the instruments (that were) used and the set objectives make this a qualitative research, since it was not intended to evaluate models or validate hypotheses or preconceived theories, in order to analyze them by mathematical methods. Among the results (that were) obtained, it was found that the difficulties related to the learning of the linear function concept are associated with the transfer through the different semiotic registers of representation that it has, especially the transfer to the algebraic and graphic registers. It was concluded that it is indeed possible to achieve an understanding of the concept of function in which students develop variational thinking through the recognition, perception and characterization of variation and change in different contexts, in order to model, describe and represent it using different semiotic registers, as proposed by the curricular guidelines proposed by the Ministry of National Education.

Keywords: Learning, linear function, didactic engineering, modeling, didactic situation.

Introduction

When persons are asked about what they consider to be proficient in mathematics, many associate it with the ability to estimate calculations and perform operations with ease, which reduces the mathematical task to an algorithmic practice, leaving aside the concepts associated with mathematical objects.

Thompson (1992), cited by Villanova, et al., (2001) points out that there is a vision of mathematics as a discipline characterized by precise results and infallible procedures, whose basic elements are arithmetic operations, algebraic procedures, geometric terms and theorems; therefore, knowing mathematics is equivalent to being able to developing procedures and identifying the basic concepts of the discipline. The conception of teaching mathematics that emerges from this vision leads to an education that puts the emphasis on the manipulation of symbols, whose meaning is rarely understood.

This vision is reflected in the results of tests such as PISA, OECD and SABER; in which Colombian students consistently get low performances, thanks in large part to the fact that the school prioritizes performing procedures and algorithms in a mechanical way, above the comprehension of concepts; that is, students know how to perform operations, but do not understand very good when to use them.

One of the mathematical concepts that has suffered most from this phenomenon of *logarithmic exercise*, by calling it in some way, has been that of mathematical function, a term that must be the subject of compulsory analysis, since the adequate understanding of it is fundamental for the study of more complex mathematical structures.

Faced with this situation Dorado & Díaz (2014) propose a reflection on the importance of the concept of function in the mathematics curriculum, because it is essential to acquire other knowledge, even in other disciplines. However, according to a study conducted by Villa, Bustamente, Berrio, & Osorio (2008), it was found that in some institutions of basic and secondary education, the teaching of the concept of function is limited to an algorithmic practice, applied to few situations that little or nothing have to do with reality.

In relation to the above, González Martín & Camacho (2005), cited by Peña & Aldana (2013), point out that “*some difficulties associated with the concept of function may be due to the great variety of representations that it has (graph, diagram of arrows, formula, table, verbal description ...); and the relationships between them*” (p.151).

Based on the above considerations, the general objective of this study is to enhance the learning of the concept of linear function with eighth grade students, through the modeling of situations taken from the context.

In correspondence with the general objective, the specific objectives are associated with each of the phases of the didactic engineering methodology. The first one, which is derived from phase 1 “Preliminary analysis”, is to know the historical, epistemological, cognitive and didactic aspects associated with the concept of function; the second one, derived from phase 2 “Analysis a priori”, is to identify the difficulties presented by students when they solve tasks of modeling situations of problems of their daily life, associated with the concept of function; the third one, derived from phase 3 “Experimentation”, is to analyze the comprehension that students reach of the concept of function through a process of modeling situations in context; and the last specific objective derived from phase 4 “A posteriori analysis”, is to validate the level of learning achieved by the students of the concept of function, by means of the comparison of *a priori* and *a posteriori* analysis.

The difficulties associated with the incorrect appropriation of the concept of function by students, a consequence to some extent of the way how it is developed in textbooks, which privileges the development of algorithmic and algebraic procedures, above the analytical meaning of the concept, together with the contributions of various related studies, have been the starting point to formulate the following research problem: How to promote the learning of the linear function concept in the eighth grade students of the Educational Institution *Nuestra Señora de los Dolores* (Quinchia Risaralda)?

In order to achieve the general objective, and therefore to answer the research question, the work was developed in different stages; first, a bibliographic review was carried out with which the state of the art could be established; then, there were developed each one of the phases of didactic engineering, as described below:

Phase 1 Preliminary analysis: A bibliographic search was carried out, about the concept of function, from its beginnings to the present time. While the historical research was carried out, there were analyzed the conceptions of a group of eleventh grade students for the analysis of the cognitive dimension, as well as the conceptions of a group of teachers for the analysis of the didactic dimension.

As a result of the preliminary analysis, there were established the following categories, which were analyzed through the actions of the students in phases 2, 3 and 4 of the didactic engineering.

Category 1:

Recognition and description of the dependency between variables.

Category 2:

Modeling of verbal, arithmetic or algebraic expressions.

Category 3:

Conversion of one semiotic representation system into another.

Category 4:

Graphic representation of a dependency relationship through a Cartesian product.

Category 5:

Domain and range of the function.

Phase 2 *A priori* analysis: In this phase, the students were offered a didactic situation in which they had to model a context situation.

Phase 3 Experimentation: In this phase, two didactic situations were proposed to the students; and then, the didactic situation developed in phase 2 was again proposed to them, in order to carry out the confrontation between the *a priori* and *a posteriori* analysis in phase 4.

Phase 4 *A posteriori* analysis and validation: In this phase of engineering, it was carried out a comparison between the *a priori* analysis, derived from phase 2; and the *a posteriori* analysis, associated with phase 3.

Finally, the conclusions were drafted; and it was written up the report of the investigation.

Theoretical perspective

Difficulties associated with the concept of function

According to research conducted by Azcárate & Piquet (1996) and Higuera (1998), cited by Castro & Díaz (2014), related to the difficulties associated with learning the concept of function, they are caused, among others, by the following aspects:

- *The poor construction of the concept carried out by the students.*
- *The lack of significant situations during their learning, which is directly related to the traditional pedagogical model used by the teacher.*
- *The kind of activities developed with the different registers of representation, which do not promote the understanding of the elements immersed in the concept.*
- *The exercise of the symbolic, which favors the mastery of algorithmic processes in problem situations where the concept of function is used, but when faced with contextualized situations, students find it difficult to solve them due to the poor understanding of elements such as the identification of the type of function, the variables, and their dependency relation (p.12 -12).*

Construction of the function concept

After verifying that the students handle the required pre-concepts, the construction of the function concept can begin. Because functions are an inclusive category of relationships, it is necessary to introduce the concept of function based on the analysis of situations that allow the study and analysis of relationships.

This process should allow to establish differences between the relationships that are not functions, from the ones that are; and (starting) from the last ones, to identify characteristics and properties taking into account the different representations.

Mathematical definition of function

The concept of function in mathematics is very important because it allows to model some phenomena, such as costs, purchases, transfers and calculations of perimeters; but above all, its application in everyday life is in the business sector, in the economic aspect, in the use of supply and demand, which are not only found in mathematical contexts but also in science contexts; functions that have the form $f(x) = ax$ and $f(x) = ax + b$ are the simplest linear models; they represent for many students the first contact with the concept of function.

For the purposes of this research, there was assumed the following concept of function, proposed by Dirichlet (1837), cited by Ugalde (2013).

If a variable «x» is related to another variable «y» in such a way that whenever a numeric value is assigned to «x», by means of a rule according to which a single value of «y» is determined, then it is said that «y» is a function of the independent variable «x». (p. 15)

Theory of didactic situations TSD

The theory of didactic situations arises under the French school of didactics of mathematics, headed by Guy Brousseau (2007), as a tool to discover and interpret the phenomena and processes linked to the acquisition and transmission of mathematical knowledge.

According to Panizza (2003) the proposed situations must include different types of responses from the students to which they are proposed, which can be of three classes: “*action, formulation or validation*” (p.10).

Situations of action: they work on the environment and favor the birth of (implicit) theories that function as models whose properties are used in practice to solve certain problems (Panizza, 2003. p.10).

In other words, they are those situations in which students must perform an action on a medium, whether material, symbolic or artificial, for which they require using implicit knowledge.

Formulation situations: they favor the acquisition of explicit models and languages; if they have an explicit social dimension, they speak of communication situations (Panizza, 2003. p.11).

It is, therefore, about situations in which the student formulates a message based on the task in question, and the recipient of that message must understand it and act on a material, symbolic or artificial medium, based on the content of that message.

Validation situations: those where the students are asked for tests and, therefore, explanations about the theories (that were) used; in addition, to make explicit the media that underlies the demonstration processes (Panizza, 2003. p.11).

Didactic contract

The didactic contract is then constituted in a system of norms; some of these, mostly generic, can be lasting; others, most of which are specific to the knowledge under study, must be defined according to the progress of knowledge.

Didactic situation

Within the situations that can be offered to the student, this theory poses didactic situations, which are intentionally constructed with the aim of having the students to acquire a certain knowledge; Brousseau (1982), cited by Galvez, G (1994) and resumed by Panizza (2003), defines them as:

A set of explicitly and/or implicitly relationships, established between a student or a group of students, a certain medium (which eventually includes instruments or objects) and an educational system (represented by the teacher), in order to get these students to appropriate a constituted knowledge, or in the process of being constituted (p.4).

A-didactic situation

Within this dynamic, another dimension is identified: the a-didactic situation, which is understood as the process in which the teacher presents students with a problem that resembles real-life situations that can be addressed through their previous knowledge and that will allow you to generate hypotheses and conjectures that resemble the work that is done in a scientific community. In other words, the students will be seen in a scientific micro-community solving situations, without the direct intervention of the teacher, with the purpose of later institutionalizing the acquired knowledge.

For Brousseau (1986), cited by Panizza (2003), the a-didactic situation term: *Designates any situation that, on the one hand cannot be dominated conveniently without the implementation of knowledge that is intended; and that on the other, sanctions the decisions taken by the students (good or bad), without intervention of the teacher, regarding the knowledge that is put into play. (p.4)*

Institutionalization

Institutionalization occurs when in a situation, the value of a procedure is recognized as a reference procedure, as if it were a formula, since in didactic situations it is necessary to identify which of the properties that are found and that should be preserved.

It is important to emphasize that institutionalization seeks to establish relationships between students' productions with the scientific knowledge inherent to the mathematical object in question; therefore, scientific knowledge should not be presented without relating to the work (that is) developed during a class.

Didactic engineering

The notion of didactic engineering was introduced in the didactics of French mathematics at the beginning of the 80s, in order to describe a way of approaching didactic work comparable to the work of the engineer (Artigue, 1995). This comparison is based on the assumption that, in order to carry out a project, engineers rely on the scientific knowledge of their domain, they accept to undergo scientific control, but at the same time, they are obliged to work on objects much more complex than those of the science; therefore, they can address problems that science cannot yet take charge of.

Didactic engineering, as a research methodology, presents among its main characteristics, that it is a research based on didactic interventions in class; that is, on the conception, realization, observation and analysis of teaching sequences.

Another characteristic that is worth noting is that validation is essentially internal, based on the comparison between *a priori* and *a posteriori* analysis (and not external validation, based on comparing the performance of experimental and control groups).

For the development of this type of research, there must be taken into account the four phases that it presents: a) Preliminary analyses; b) Conception and *a priori* analysis of didactic situations; c) Experimentation; d) *A posteriori* analysis and evaluation.

Preliminary analysis

According to Artigue (1995), the preliminary analysis considers three fundamental dimensions within didactic engineering.

- The epistemological dimension, associated with the characteristics of mathematical knowledge in play.
- The cognitive dimension, associated with the cognitive characteristics of the audience to which the teaching is directed.
- The didactic dimension, associated with the characteristics of the functioning of the teaching system (p 40).

Conception and *a priori* analysis

Traditionally, this *a priori* analysis comprises a descriptive and a predictive part; it focuses on the characteristics of an '*a-didactic*' situation, which has been designed and will be proposed to the students.

In this situation, it is analyzed what could be learnt by a student, depending on the possibilities of action, decision, control and validation that are available once put into practice, when working independently of the teacher. In addition, possible behaviors are envisaged; and it is aimed to demonstrate how the analysis (that is) carried out, allows to control its meaning and to ensure that, if the expected behaviors are produced, they be the result of putting into practice the knowledge pretended by learning.

Experimentation

It is the phase of didactic engineering, where the researcher has direct contact with a certain population of students who are under investigation.

For the development of this phase, the researcher must make explicit the objectives and conditions of conducting the research to the students who will participate in the experimentation; in addition, to establish a didactic contract; and finally, to apply the research instruments designed according to the research problem.

A posteriori analysis and validation

The experimentation phase is followed by *a posteriori analysis*, which is based on the analysis of the set of data collected throughout the experimentation, including the observations of the teaching sequences and the productions of the students in the classroom, or outside it. These data are often completed with others obtained through questionnaires, individual or small groups interviews, applied at different times of teaching, or during their course. And as already indicated, in the comparison of the two analyses, the *a priori* and the *a posteriori*, it is based the validation of the hypotheses formulated in the research.

Materials and methods

Taking into account that this research was not intended to evaluate models, or validate hypotheses or preconceived theories to analyze them with mathematical methods, it is then a qualitative research, as proposed by Sampieri, Collado, & Lucio (2014) when defining the characteristics of qualitative research; they mentioned that it “*is based on an interpretative perspective centered on the understanding of the meaning of the actions of living beings, mainly humans and their institutions*” (p 358 - 359).

The population was constituted by 35 students of eighth grade, of the educational Institution *Nuestra Señora de los Dolores*, of the municipality of Quinchía (Risaralda province). This group was selected because according to the basic learning rights emanating from the MEN in 2015, the eighth grade begins the study of the linear and quadratic function, oriented towards its understanding and application in situations of variation.

The research (that was) carried out was descriptive in that it described the productions and conceptions of students when they perform tasks related to situations that can be modeled by a linear function. As a main technique, the task-based interview proposed by Davis (1984), cited by Caicedo & Díaz (2014).

During the development of the research, four interventions were carried out with students; in the first one, the didactic situation was developed; in the second and third intervention, two didactic situations were worked on; and in the fourth one, the situation was applied again to didactics, in order to carry out the validation of the learning achieved through the confrontation between *a priori* and *a posteriori* analysis. Each session lasted approximately 3 hours of classroom work, supplemented by semi-structured interviews with the students.

Results

Confrontation category 1

Recognition and description of the dependency between variables

A priori analysis

Graph 1 Response of student A24 to a didactic situation of action

a. ¿Cuál es el plan más favorable si se hablan menos de 120 minutos en el mes?
es el de 30

Porque?
es lo mas favorable por minuto

b. ¿Cuál es el plan más favorable si se hablan más de 200 minutos al mes?
el plan mas favorable es de 75000

Porque?
por que es lo mas favorable por mes.

A posteriori analysis

Graph 2 Response of student A24 to a didactic situation of action

a. ¿Cuál es el plan más favorable si se hablan menos de 120 minutos en el mes?
la empresa B

Porque?
Porque al multiplicar 750 x 720 nos da 78.000 en cambio
la A al multiplicar 720 x 50 y esto mas 75.000 nos da 27.000

b. ¿Cuál es el plan más favorable si se hablan más de 200 minutos al mes?
la empresa A

Porque?
al multiplicar 200 x 50 más 75.000 nos da 75.000 mientras que
la B al multiplicar 200 por 750 nos da 30.000

Source: own production

Confrontation: In phase 2 (*a priori*), it was found that a significant number of the student population do not internalize the concept of dependency relationship between variables, so it is difficult to understand that in a given situation in which there are two variables related among themselves, any variation suffered by one of the variables, called the independent one, determines the variation suffered by the other variable, called the dependent one.

When comparing the results obtained in phase 2 (*a priori*) with those obtained after the experimentation phase, in which there were modeled different situations related to the context; and when analyzing the performance of the students when developing a didactic situation of action, in which it was necessary to recognize the relationship of dependence between the variables *number of minutes* and *bill cost*, it could be seen that the students actually recognized the dependency relationship, and identified which of the companies was more favorable, and that this favorability depended directly on the number of minutes consumed.

Like the student A24, 31.7% of the students recognized the dependency relationship and correctly solved the task, which represents a 20% of advance, compared to the performance of the same group in phase 2.

Confrontation category 2

Modeling of verbal, arithmetic or algebraic expressions

A priori analysis

Graph 3 Response of student A18. Didactic situation of formulation

	PLAN A.	total		PLAN B.	total
Minutos	100	20.000		100	15.000
costo	$\times 50$	5000		$\times 150$	
impuestos	15.000			0	

A posteriori analysis

Graph 4 Response of student A18. Didactic situation of formulation

PLAN A.	PLAN B.
$(x \cdot 50) + 15.000 = Y$	$(x \cdot 150) = Y$

Confrontation: As it can be seen in the *a priori* analysis, most students, like A18, could not propose a valid arithmetic process able to model the dependency relationship between the variables.

Student A18 was one of the few who particularly dared to propose a valid arithmetic expression to determine the cost of each plan for any number of minutes; however, he failed to transcend the algebraic record, and thus generalize the dependency relationship in the situation proposal.

When comparing the number of students who proposed an algebraic expression that generalizes the procedure and models the situation, a significant increase was observed in percentage terms, from 2.8% of students in phase 2 (1 of 35), to 45.7% in phase 3 (16 of 35).

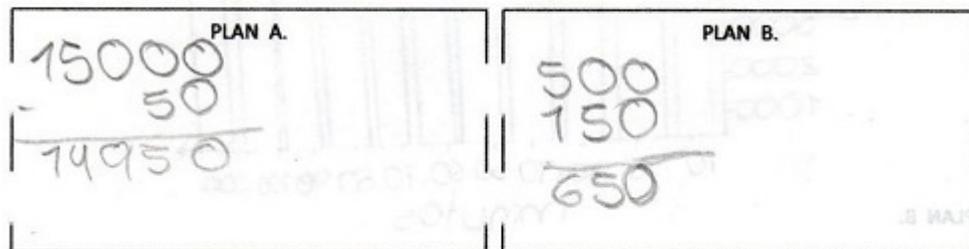
Confrontation category 3

Conversion of one semiotic representation system into another.

A priori analysis

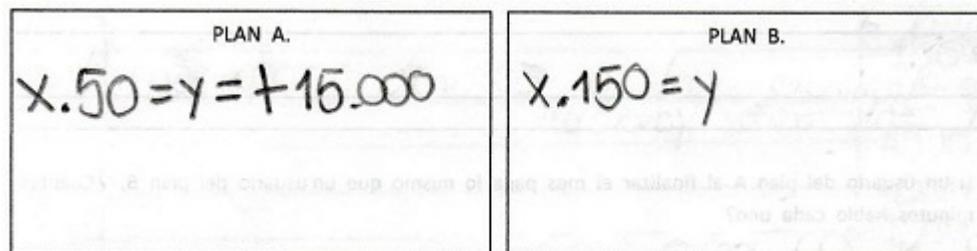
Graph 5 Response of student A18. Transfer of the verbal register to the algebraic one. Didactic situation of formulation

Representa la descripción anterior a través de una expresión matemática que permita calcular el costo de la factura para cada uno de los planes a partir de la cantidad de minutos gastados.

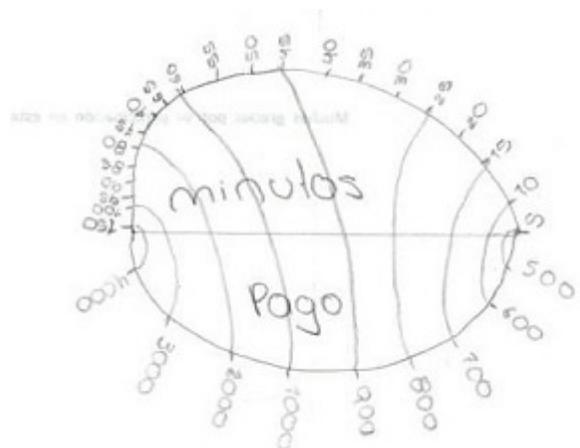


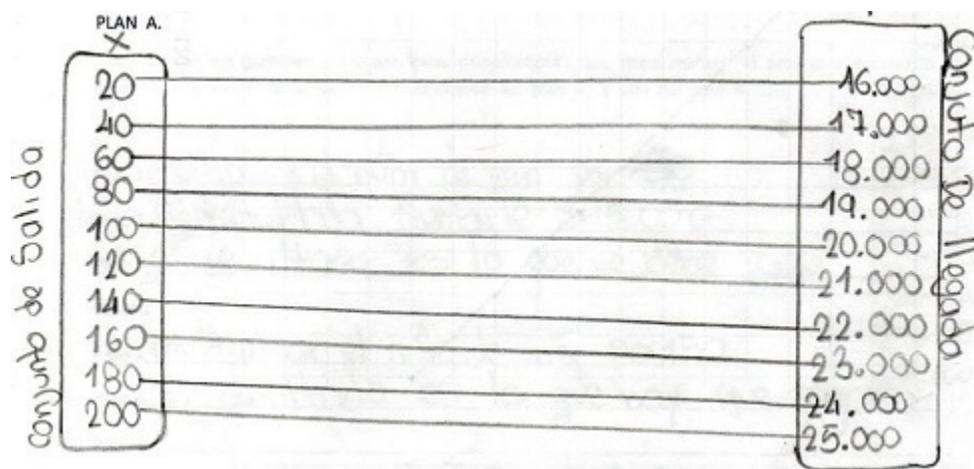
A posteriori analysis

Graph 6 Response of student A18. Transfer of the verbal register to the algebraic one.



Graph 7 Response of student A18. Way to represent a function. Didactic situation of validation



Graph 8 Response of student A18. Way to represent a function. Didactic situation of validation

Confrontation: The *a priori* analysis showed that when making conversions from one system to another, there is a lack of verbal and arithmetic tools to describe the process that allows determining an element in the arrival set from another element in the output set.

A significant number of students were not able to pose an arithmetic expression, even with errors that would approach the required procedure; and only one of the 35 managed to model the situation in context through an algebraic expression that generalizes a measurement pattern in the process, in order to establish the relationship between the number of minutes and the cost of the bill.

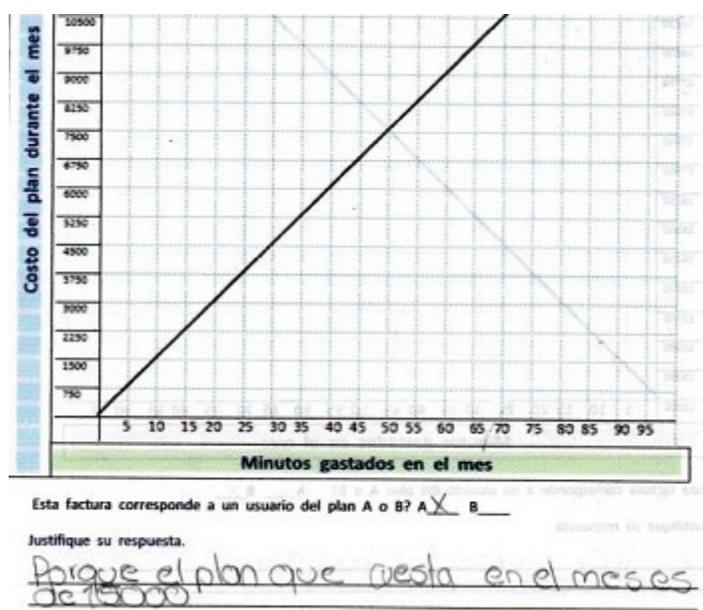
When comparing the performance of students in phase 2, “*a priori* analysis”, with the performance of the same students in phase 4, “*a posteriori* analysis”, it can be validated that there was a notable improvement in the performances of a significant part of the group, especially regarding the transfer through the different semiotic registers of representation of the function, which is basically summarized in the refinement of the capacities to pose algebraic expressions that generalize a dependency relation, the use of the tabular register as a valid form of representation of a relationship of dependence, and a better understanding of the relationship between the variables through a Cartesian product.

Confrontation category 4

Graphic representation of a dependency relationship through a Cartesian product

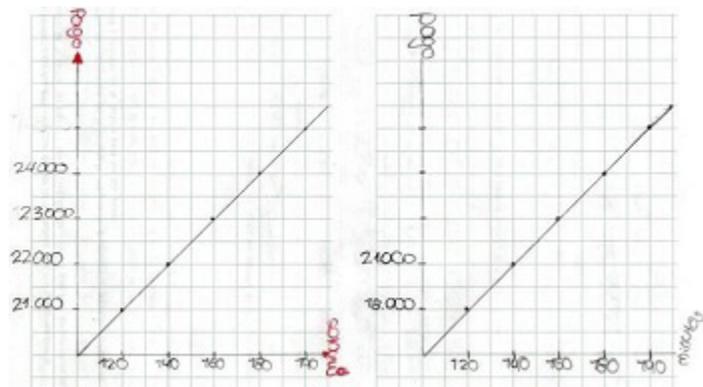
A priori analysis

Graph 9 Response of student A28. Didactic situation of validation



A posteriori analysis

Graph 10 Response of student A28. Didactic situation of validation



Confrontation. The *a priori* analysis of category 4 showed that 77% of the students are not familiar with the graphic representation system, which is the reason why it is difficult for them to recognize in a graph the dependence relation between two variables.

This fact makes it impossible for them to recognize the graph that corresponds to a certain function, since they focus their attention on the values present in the axes, but do not give greater importance to the Cartesian product that represents the relationship between the variables.

Like the A28 student, the group in general demonstrated significant advances in the management of the semiotic register of graphic representation, which is observed in an increase of 23% that recognized a dependency relationship through a graph in phase 2, until a 51% that recognizes and represents this relationship through a Cartesian product after completing phase 3.

Confrontation category 5

Domain and range of function

A priori analysis

Graph 11 Response of student A16. Didactic situation of validation

e. ¿Cuántos minutos se habla en cada plan si la factura llega de \$45000?

Se habla 600 minutos

Porque?

La factura ha sido el valor del mismo precio por que en ese caso habla más minutos por el plan.

A posteriori analysis

Graph 12 Response of student A16. Didactic situation of validation

En la empresa A habla 600 minutos y en la B 300 minutos

Porque?

Confrontation: The *a priori* analysis of category 5 showed that only 14% of students understood that, given a relationship of dependency posed through a functional relationship, obtained from the modeling of a situation related to the context, there are some values of both the domain and the range that do not make sense.

A significant number of students, like student A16, have correctly understood that in spite of being two linear functions that model the situation, there is a value of the rank that is common to both, and that such rank is related to different domains in each of the functions; this fact assumes that the students internalized that there are values which may or may not have sense when they are extracted from a situation of the context modeled through a function.

The comparison of the *a priori* analysis with the *a posteriori* one (corresponding to category 5), showed an increase from 14% in phase 2, to 57% after phase 3, which is really significant.

Discussion

From the confrontation between the *a priori* analysis with the *a posteriori* analysis, it remains as a reflection that the learning of a mathematical object, the function in this case, becomes much more significant when there are planned didactic sequences related to the context of students, in which they can use their own experience to solve the problems that this situation may pose.

In the same sense, modeling as a process for the teaching of mathematics allows students to transcend the traditional algorithmic practices associated with the concept of function, since it is them who pose the procedures and algebraic expressions that allow them to generalize the didactic situation; likewise, meaning is given to the use of semiotic registers of representation, since they are used because of the need to represent the sets involved in a dependency relationship, and not only to satisfy a requirement imposed by the teacher.

As stated by the MEN (1998), problem solving as a strategy to model situations that give meaning to mathematical objects “provide the immediate context in which mathematical work makes sense, to the extent that the situations addressed are linked to daily experiences and, therefore, are more meaningful for students” (p., 52).

Conclusions

After the confrontation between the *a priori* analysis and the *a posteriori* analysis, it was concluded that it is indeed possible to achieve a learning of the linear function concept in which the students develop variational thinking through the recognition, perception and characterization of the variation and the change in different contexts, in order to model it, describe it and represent it using different semiotic registers, as proposed by the curricular guidelines proposed by the MEN. Such learning was evidenced through:

Definition of a function concept using proper language, such as the relationship between two sets according to which each element ‘*a*’ of a set A is related to an element ‘*b*’ of a set B, because each variation or change that A suffers determines a variation or change in B.

Construction of algebraic or arithmetic expressions that model a situation and offer a way to represent a dependency relationship using mathematical language, recognizing in the generality of algebra a powerful tool for modeling situations or phenomena in which there is variation.

Use of the graphic register to represent the dependency relationship between two variables, and analyze the variation that occurs in each element of the arrival set from the variation suffered by each element of the output set, which translates into growth, decrease and increase.

Use of the criterion of uniqueness to differentiate a function from a relation, from the analysis of the tabular register and the vertical line criterion for the analysis of the graphic register.

Understanding the meaning of the domain and the range of a function from a situation related to the context; that is, it is understood and justified when the values assigned to the output set have or do not have meaning, given the didactic learning situation; in addition, they analyze the coherence of the elements of the arrival set, and when the elements seem to have no meaning, as in the case of negative values, they can explain the meaning of these values.

Regarding modeling as a process for the learning of mathematical objects, it was concluded that the use of situations related to the context allowed the students to understand them better; and therefore, to give them meaning, thus facilitating the transfer through the different semiotic registers. of representation, as mentioned by Villa & Mesa (2007), when they suggest that “*Modeling as a learning strategy for mathematics provides a better understanding of mathematical concepts, while allowing to become a motivating tool in the classroom*” (p.10).

According to Campeón & Aldana (2016), this fact allows students to assume variation as something natural, if it is related to the context; however, the use of artificial situations and excessive formalism makes it totally abstract, and most students are not able to understand it.

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